Lecture 10: Power rankings

Skidmore College, MA 276

Goals

- Power rankings in sports
- Models for paired comparisons
- Tools: Elo, Bradley-Terry

Set-up:

Let's rank NBA teams at this exact moment in time. Judging criteria: Who would win a game played tomorrow on a neutral site?

Rank	Team
1	Golden State
2	San Antonio Spurs
3	OKC? Cleveland?

Paired comparisons

In a league with n_t number of teams, there are $n_t!$ possible allocations of power rankings. Ultimately, each rank comes down to a set of decisions called **paired comparisons**, where each player or team is compared to another player or team.

Ex: If $P(OKC > Cleveland) > 0.5$, then OKC ranks 3 (assuming OKC also better than all other teams besides Golden State, San Antonio.)

Note: $P(OKC > Cleveland) > 0.5$ is the probability of Oklahoma City beating Cleveland

What assumptions are we making in doing several paired comparisons?

Idea of paired comparisons

One participant versus another (see more here):

1. Players, teams in sport
2. Consumer products in market research
3. Images in psychology

Most common paired comparison model: **Bradley-Terry (BTM)**

Notation, BTM

- Players (or teams) i and j
- Assume $P(i \text{ beats } j) = \frac{\alpha_i}{\alpha_i + \alpha_j}$
- α_i and α_j reflect player **abilities**
  - $\alpha_i > 0$ and α_j 0
- Odds $(i \text{ beats } j) = \frac{P(i \text{ beats } j)}{P(j \text{ beats } i)} = \frac{\alpha_i/(\alpha_i+\alpha_j)}{\alpha_j/(\alpha_i+\alpha_j)} = \frac{\alpha_i}{\alpha_j}$

Example, BTM:

Rank	Team	α
1	Golden State	5.3
2	San Antonio Spurs	4.7
3	OKC	2.9

Estimate

1. P(Golden State > OKC)
2. Odds(Golden State > OKC)
3. P(San Antonio > Golden State)
4. Odds(San Antonio > Golden State)

Notation, BTM

- logit(P(i beats j)) = log(Odds (i beats j)) = log(α_i) - log(α_j) = λ_i - λ_j
  - $\lambda_i = log(\alpha_i)$ for all i
  - $\alpha_i = e^{\lambda_i}$ for all i

Example, BTM:

Rank	Team	λ
1	Skidmore	1.2
2	Vassar	0
3	RIT	-0.9

Questions

1. P(Skidmore $>$ RIT)
2. What does it mean to have a λ of 0?

How to find BTM parameter estimates?

```r
library("BradleyTerry2"); library(mosaic)
data("baseball", package = "BradleyTerry2")
head(baseball)
```

```
##   home.team away.team home.wins away.wins
## 1 Milwaukee   Detroit         4         3
## 2 Milwaukee   Toronto         4         2
## 3 Milwaukee  New York         4         3
## 4 Milwaukee    Boston         6         1
## 5 Milwaukee Cleveland         4         2
## 6 Milwaukee Baltimore         6         0
```

The model

```
baseballModel1 <- BTm(cbind(home.wins, away.wins), home.team, away.team,
  data = baseball, id = "team")
msummary(baseballModel1)
```

```
## Coefficients:
##                Estimate Std. Error z value Pr(>|z|)
## teamBoston       1.1077     0.3339   3.318 0.000908 ***
## teamCleveland    0.6839     0.3319   2.061 0.039345 *
## teamDetroit      1.4364     0.3396   4.230 2.34e-05 ***
## teamMilwaukee    1.5814     0.3433   4.607 4.09e-06 ***
## teamNew York     1.2476     0.3359   3.715 0.000203 ***
## teamToronto      1.2945     0.3367   3.845 0.000121 ***
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 78.015  on 42  degrees of freedom
## Residual deviance: 44.053  on 36  degrees of freedom
## AIC: 140.52
##
## Number of Fisher Scoring iterations: 4
```

Where's Baltimore?

Next steps

```r
BTabilities(baseballModel1)
```

```
##              ability      s.e.
## Baltimore 0.0000000 0.0000000
## Boston    1.1076977 0.3338779
## Cleveland 0.6838528 0.3318764
## Detroit   1.4364084 0.3395682
## Milwaukee 1.5813559 0.3432557
## New York  1.2476178 0.3358606
## Toronto   1.2944851 0.3366691
```

Next steps

```
exp(BTabilities(baseballModel1))
```

```
##              ability     s.e.
## Baltimore 1.000000 1.000000
## Boston    3.027380 1.396373
## Cleveland 1.981497 1.393581
## Detroit   4.205564 1.404341
## Milwaukee 4.861543 1.409529
## New York  3.482038 1.399144
## Toronto   3.649117 1.400276
```

Find estimated probability that (i) Boston defeats Cleveland and (ii) Baltimore defeats Boston

Home field advantage

```
baseball$home.team <- data.frame(team = baseball$home.team, at.home = 1)
baseball$away.team <- data.frame(team = baseball$away.team, at.home = 0)
baseballModel2 <- update(baseballModel1, formula = ~ team + at.home)
msummary(baseballModel2)
```

```
## Coefficients:
##                Estimate Std. Error z value Pr(>|z|)
## teamBoston       1.1438     0.3378   3.386 0.000710 ***
## teamCleveland    0.7047     0.3350   2.104 0.035417 *
## teamDetroit      1.4754     0.3446   4.282 1.85e-05 ***
## teamMilwaukee    1.6196     0.3474   4.662 3.13e-06 ***
## teamNew York     1.2813     0.3404   3.764 0.000167 ***
## teamToronto      1.3271     0.3403   3.900 9.64e-05 ***
## at.home          0.3023     0.1309   2.308 0.020981 *
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 78.015  on 42  degrees of freedom
## Residual deviance: 38.643  on 35  degrees of freedom
## AIC: 137.11
##
## Number of Fisher Scoring iterations: 4
```

Compare the fit of these two models:

```
AIC(baseballModel1)
```

```
## [1] 140.5186
```

```
AIC(baseballModel2)
```

```
## [1] 137.108
```

```
exp(baseballModel2$coeff)
```

```
##    teamBoston teamCleveland   teamDetroit teamMilwaukee   teamNew York
##      3.138681      2.023228      4.372597      5.050842       3.601464
##   teamToronto       at.home
##      3.770133      1.352914
```

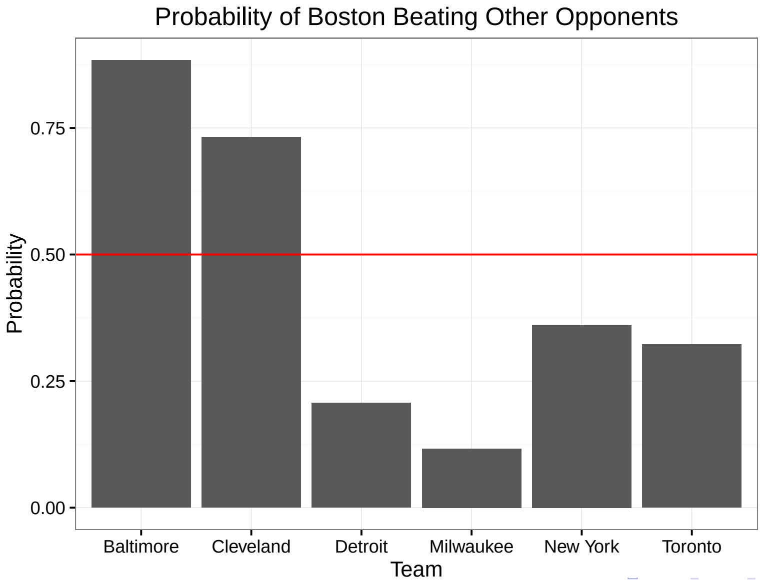
Interpret the effect of HFA in baseball.

Challenges and final thoughts

- Importance of data formatting for BTM
- Links to Elo
  - $\alpha_i = e^{S_i/k}$
  - k a sport-specific scale factor
  - S_i another team-level skill factor
  - Iterative process (can update after a game)
- Similarity to other systems (log 5, item-response)

Fun

```
p
```


Probability of Boston Beating Other Opponents
Probability
0.75
0.50
0.25
0.00
Baltimore
Cleveland
Detroit
Milwaukee
New York
Toronto
Team